Oil Refinery Maintenance as a Strategic Decision

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Abstract

In this thesis I construct a model of interaction of refinery firms which decide on their maintenance and production levels. Both of these choices are considered strategic variables in my model as they affect the probability of failure of refinery machinery. Using game theory tools and simulations, I first analyze the competition of refineries in static one-shot game and identify their strategies that lead to a unique equilibrium outcome. Further, to improve the assumptions on maintenance and production choices, I model a simple 2-period dynamic game which is later extended with an introduction of uncertainty. Dynamic model allows answering the question of whether it is beneficial to take up maintenance at all. In each model simulations are used to simplify the derivations and uncover the dependence of results on parameter values. The results of static model suggest that refineries should produce at a low level with a less need for maintenance. Dynamic model suggests that it is better to skip the production in the first period and take up maintenance instead. However, after introducing uncertainty the results of dynamic model depend on beliefs of refinery firms about future demand.

Acknowledgments

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Introduction

Generally crude oil can not be consumed as a fuel or lubricant in its initial form. Changing its structure to make it consumable is the middle step of the whole process. This significant position is held by refinery industries which link crude oil extraction to sales of final refined products in markets. The specificity of this position can be seen in its many differences from other parts of the oil sector.

One of the most important of these differences is that refinery firms have to answer more production questions in decision-making and therefore have a larger scope for optimization. Refineries are constantly falling under the pressure of market demands due to the fact that the basic refining process allows the firms to extract only a fixed amount of different products from a unit of input (crude oil). So in order to satisfy the market demand, firms are compelled to apply more complex processes, which demand costly investments[1].

This pressure increases with the need for frequent maintenance works on refining machinery and equipment. In the majority of cases these kinds of works incur not only costs but also demand the stoppage of production for the period of maintenance. With growing demand on petroleum products, refining firms have a strong incentive to produce at maximum production rates and capacity possibilities. Refining facilities process several hundred thousand barrels of crude oil per day. Taking this into consideration, one understands that the supply side of the market and the prices of final products significantly depend on capacity possibilities, unexpected refinery outages and decisions on frequency of maintenance works. It is clear that choosing the "right" time for stoppage is a challenge for refinery firms.

[1] For more detailed information about refining technologies see Manzano (2005).

An importance of maintenance decisions can also be seen from the perspective of complexity of refinery systems: the sophistication of the refining process has grown since the beginning of technological revolution due to competition among firms and the need for frequent changes in the fractions of different refined products to satisfy the constantly changing demand. As a result, vulnerability of refinery system and machinery to breakdowns (even in small areas) has greatly increased and brought up new reasons for increasing frequency and quality of machinery maintenance.

There were numerous explosions and fires at oil refineries in different countries during the past decades, majority of which happened due to the failure in machinery system or leakages. For example, British Petroleum oil refinery explosion in Texas (March 2005) led to fifteen deaths, more people were wounded. This event led to huge losses for society, environment and for the refinery firm itself. The reports said that the explosion happened due to a breakdown in the system[2]. Another example is the fire in Bombay High oil field in India (2005), when four people were killed and the platform itself was completely destroyed in the fire. The reason for this event was a leakage in a naphtha pipeline[3]. The maintenance decisions have become crucially important in preventing these kinds of disasters and in safer cases – huge costs to the firm.

In my thesis I analyze decisions of competing refinery firms on quantity supplied to the market and maintenance works in game theory context in order to answer the questions of how much effort it is optimal to put in taking up maintenance and how to combine this choice with an optimal choice of production level. Maintenance is assumed to be a strategic decision for a firm as it directly affects the probability of machinery failure. Choice of maintenance will be first presented as a choice of percentage of the whole refining machinery to be repaired or renewed in a static model, later presented as a binary choice in a dynamic model. Making reasonable assumptions about the

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[2] URL: http://www.popularmechanics.com/technology/industry/1758242.html
[3] URL: http://news.bbc.co.uk/2/hi/south_asia/6319459.stm

behavior and choices available to firm, I will first model the problem of one firm and its actions assuming no competition. Later, using game theory I will model and analyze the interaction of two refinery firms in particular situations in order to identify their strategies and responses to the actions of rival firm. Numerical simulations will be used in all analyzed problems both for simplification of derivations and quantitative evaluation of results.

Generally, known from MOL experience[4], refinery firms do not optimize, but iterate their profits using different combinations of strategic variables and choosing the ones yielding the highest value for the profits. This particular way of maximization can sometimes be useful; however iteration does not necessarily yield maximizing results and there is a scope for optimization.

Little research has been devoted to an optimal choice of maintenance and optimization of production shut down. Muehlegger (1997) in his doctoral thesis models a choice of oil refinery between output and maintenance to find out whether change in ownership affects the probability of outage. He addresses the problem of unexpected outages and their possible correlation to the immediate price increases, which can create an incentive for aimed (and sometimes unnecessary) shutdowns. Fudenberg and Tirole (1984) analyze similar to Muehlegger`s (1997) two-step game with capacity and entry decisions. A game of simultaneous and sequential choice of road maintenance and tolling decision for competing road firms is analyzed in de Palma et al. (2006). Even though there are major differences between road and refinery sectors, their model is able to identify optimal maintenance levels and together with Muehlegger`s (1997) model will serve as a basis for a static model analyzed in my thesis.

As a science, game theory can be considered "young" in comparison with other branches of math and economics. The main concept of game theory – Nash Equilibrium – has been introduced less than 60 years ago. Nevertheless it has already created a lot of different branches in itself

[4] From a personal discussion with Mr. Györfi János – SCM Optimization Manager of MOL Hungarian Oil & Gas Company.

(Evolutionary, Experimental Game Theory, etc.) and is widely used in different kinds of researches. I chose game theory as a tool for this research because it proved to be a powerful and convenient tool for analyzing and predicting actions of individuals and firms (as entities controlled by individuals). With the help of classical game theory, I will model the interaction of rational self-regarding refinery firms, which (according to this theory) will arrive at their equilibrium choices – a state which appears as a result of best response of each firm to the actions of others.

This thesis is constructed as follows: Chapter 1 develops a static model of interaction of refinery firms with an assumption about simultaneous choice of maintenance and production levels and identifies equilibrium strategies of refinery firms; Chapter 2 repeats the same analysis for a dynamic model with different assumptions about the choices available to refinery firms and later extends the model with uncertainty about demand.

Chapter 1: Static Model and One-shot Games

In the first chapter I concentrate on building a static refinery model following Muehlegger (1997) and de Palma et al. (2006). I apply it to simultaneous and sequential one-shot games by introducing two competing refinery firms and specific demand function. Even though the analysis of a static model has its drawbacks (which will be discussed later) it will help to understand the actions of refinery firm under specific conditions. The chapter begins with the introduction to a more detailed refinery process and complicated choices which the firms face. It then continues with a step-by-step construction of interaction of refinery firms.

1.1 Oil Refinery Process in Detail

The choice of the crude oil type is the first step of refinery process. The price of crude oil differs mainly according to its density (light, medium or heavy), content of sulfur (sweet or sour) and acids. Light sweet crude is processed more easily and yields a greater amount of light products. Hence, this type of crude oil is more expensive due to an increased demand for lighter fuels during the last decades. Acid content is important in the sense that corrosion of refinery equipment increases with its level[5].

Initially the crude oil has to be separated by distillation into different parts, which are later improved with different refining technologies to match the quality of market demand. Refinery firms have to adjust the supply of different types of products to seasonality of their consumption. This

[5] See Manzano (2005) for more detailed information.

adjustment is done either through the choice of crude oil type or technology (or both) subject to capacity constraints of the firm.

Refining machinery and equipment have become much more sophisticated since the beginnings of the refinery industry. The reason for this sophistication is technological innovations that introduced new possibilities for firms to quickly adapt to market demand and increase efficiency. On the other hand, these new options came with greater vulnerability of refinery systems to breakdowns even in small areas. Such incidences as leakage or machinery failure could lead to fires and explosions with tragic consequences. Hence, the frequent maintenance of refining machinery and equipment works are necessary to avoid the costs of failure.

Maintenance decisions become strategic when a refinery firm chooses the time, frequency, length and quality of maintenance. Due to the changes in demand (especially seasonally) and competition these choices along with the choice of output determine the profits of the refineries. Optimization of maintenance is thus crucial for a survival of the firm.

Maintenance works in reality could be taken up simultaneously with production, however majority of works demand stoppage of production because of complex interconnection of all parts of refining machinery. With static model it is impossible to put restrictions that would allow accounting for stoppage of production. In this sense this model is less realistic than dynamic one; however it allows defining a unique equilibrium and greatly simplifies the analysis.

1.2 Simulation Method

Simulation methods have become very popular in all branches of economics. Even though they are used more often in empirical researches, here I use simulation with base values of parameters in order to simplify burdensome derivations and arrive at an equilibrium outcome. Simulations are performed using Wolfram Mathematica software[6] and with its help I analyze the model and variations of equilibrium choices and profits with all parameters.

The following numbers will be used as base values for simulation:

K	M	F	c	g	a	b
1000	100	2000	3	4	10	200

Table 1. Base values of parameters used for simulations in static model.

Simulations with other values are performed in Appendix (p. 37-38) for comparison. Capacity limit and maximum maintenance level for each firm are assumed to be equal during simulations; however in derivations of optimal strategies and equilibrium, I use separate parameters for each firm in order to obtain a general solution, which is then simplified with assumed base values.

1.3 Single Firm Analysis

Even though refinery firms produce fixed amount of heterogeneous output from one unit of crude oil depending on technology and the type of crude oil, here I will assume that refinery firms produce single unit of homogeneous output q from a homogeneous input of a unit of crude oil (that is, the technology rate is 1:1) subject to capacity constraint K_i. These assumptions simplify the analysis in the sense that refinery firms cannot affect their market supply and profits by choosing

[6] Mathematica ver. 5.0 - the product of Wolfram Research, Inc.

different types of crude oil or technologies. So the choices of refinery firms are limited to the choices of homogeneous output and maintenance[7].

The cost functions for the two variables of main interest are given respectively by:

$$c(q) = c \cdot q$$
$$g(m) = g \cdot m^2$$

In my analysis I assume that capacities of refineries affect the choices of output through the probability of failure function and do not affect the cost function, thus allowing producing with constant marginal cost.

The probability of failure φ for a refinery firm is defined as a function of q (quantity produced) and m (quality of maintenance) with cost $\boldsymbol{F}$ incurred in case of failure. It is reasonable to assume that q and m independently affect φ (even though in reality their effects can be somehow correlated), so that the probability function is represented by a linear function of two increasing and convex probability functions:

$$\varphi(q,m) = \gamma_q(q) - \gamma_m(m)$$
$$\gamma_q{}'(\cdot), \gamma_m{}'(\cdot) > 0 \ ; \ \gamma_q{}''(\cdot), \gamma_m{}''(\cdot) \geq 0$$

In cases of optimal choice derivation a simple linear probability function $\varphi(q,m) = \alpha q - \beta m$ will be used instead of the general form, with $\alpha = \dfrac{1}{K}$; $\beta = \dfrac{1}{M}$, where K is the capacity limit of refinery firm and M is the maximum possible maintenance level[8]. With this construction the effect of production level on probability of failure reaches 100% as production level approaches capacity

[7] Relaxing these assumptions will not change the outcome of the analysis. It is possible to redefine: $q = \sum_j q_j$, where *j* is the number of different types of products. The result will be the same since the analysis does not involve upgrading technologies that would change the fractions of output yields.

[8] *M* can be thought of as a maintenance of the whole refining machinery and equipment, i.e. choosing maintenance level *M*/2 means maintaining half of the refinery system. During simulations *M* will be assumed to be equal to 100 and thus the choice of maintenance will be basically a choice of percentage of refining machinery to be renewed or repaired.

limit. It is obvious that $\varphi(\cdot)$ decreases with K and increases with M (the more complex is the refining machinery, the more danger there is for a failure in one of its parts). These assumptions also assure that probability of failure stays in the zero-one interval with an additional condition that $\frac{q}{K} > \frac{m}{M}$ (which is satisfied for all following results).

Assuming there are N firms operating on the market, the price is determined from total output supplied by all firms: $p = p(Q)$ and the problem of the firm i is:

$$\max_{q_i, m_i} \pi_i = [p(Q)q_i - c(q_i)] \cdot (1 - \varphi(q_i, m_i)) - F \cdot \varphi(q_i, m_i) - g(m_i)$$
$$\text{subject to: } q_i \le K_i \text{ , } m_i \le M_i \text{ and } Q = \sum_{j=1}^{N} q_j \tag{1}$$

Optimal simultaneous choice of output and quality of maintenance can be found from the first order conditions for the problem:

$$\frac{\partial \pi_i}{\partial q_i} = \underbrace{[q_i p'(Q) + p(Q) - c'(q_i)] \cdot (1 - \varphi(q_i, m_i))}_{\text{direct effect (increase in } \pi \text{ due to an increase in output produced)}} - \underbrace{- \frac{\partial \varphi(q_i, m_i)}{\partial q_i} \cdot [p(Q)q_i - c(q_i)] - F \cdot \frac{\partial \varphi(q_i, m_i)}{\partial q_i}}_{\text{indirect effect (decrease in } \pi \text{ due to an increase in probability of failure)}} = 0 \tag{2}$$

$$\frac{\partial \pi_i}{\partial m_i} = \underbrace{- \frac{\partial \varphi(q_i, m_i)}{\partial m_i} \cdot [p(Q)q_i - c(q_i)] - F \cdot \frac{\partial \varphi(q_i, m_i)}{\partial m_i}}_{\text{this effect is positive due to decrease in probability of failure}} - g'(m_i) = 0 \tag{3}$$

In this part of analysis demand will be treated as fixed at high enough level (so that whatever is produced by the firm is sold), hence the refinery is a price-taker. Using assumed functional forms of production and maintenance costs and probability of failure function, one can find an optimal choice of output and maintenance for refinery firm as functions of fixed price p^f and costs. Graph G28 in Appendix shows how profit function changes with main variables of choice.

Solving first order conditions simultaneously optimal choices are defined as (see A1 in Appendix):

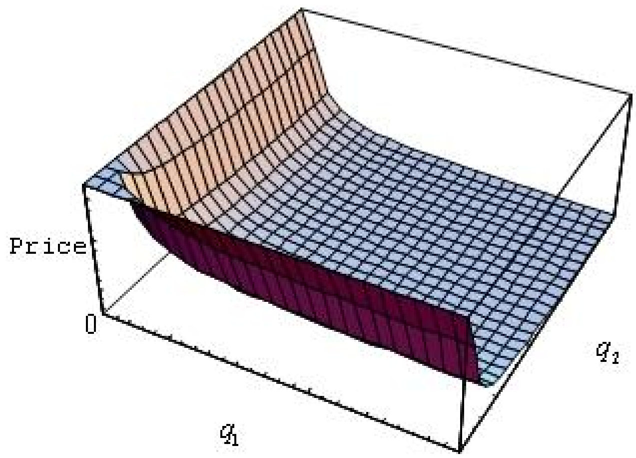
$$q_i^* = \frac{K_i(p^f - c)(F + 2gM_i^2) - 2FgM_i^2}{(p^f - c)(4gM_i - K_i(p^f - c))} \quad ; \quad m_i^* = \frac{M_i(K_i(p^f - c) + F)}{4gM_i^2 - K_i(p^f + c)}$$

An increase in capacity leads to an increase both in optimal output and maintenance levels. After differentiating both results with respect to M_i it is obvious that both q_i^* and m_i^* decrease with maximum possible maintenance level. Using reasonable assumption about an increase in optimal choice of maintenance with an increase in failure (and implied necessary conditions for it), the necessary condition for q_i^* to decrease with F is: $2gM_i^2 > K_i(p^f - c)$[9] (see A1 in Appendix).

1.4 Duopoly Analysis

This part of the Chapter deals with the interaction of two independent competing firms in a world of perfect information. Refinery firms choose quantities that all-together define the final price; hence the competition is a la Cournot. Price is determined according to the following functional form: $p(q_1, q_2) = a + \frac{b}{q_1} + \frac{b}{q_2}$, which reflects the weak response of price to supply changes if quantity supplied is large enough and strong response if supply is on low level. This function has a constant elasticity coefficient of minus one and is depicted in Graph 1.

[9] This condition holds for assumed base values of parameters unless fixed price is incredibly high.



Graph 1. Inversed demand function (price function)

Competition of refineries will be constructed first under simultaneous choice of output and maintenance by firms, followed by sequential move game.

1.4.1 Simultaneous Move Competition

Under this construction of the problem refinery firms simultaneously choose their output and maintenance levels. The equilibrium outcome can be found by maximizing the profit function (1) with respect to q and m. The resulting first order conditions are given in (2) and (3) assuming that $N = 2$. Each firm's optimal choice will depend on the choice made by a competing firm. It can be seen from (2) that optimal output for firm 1 is defined as a reaction function $q_1^*(q_2(m_2), m_1)$. Solving (3) for optimal maintenance level one obtains reaction function $m_1^*(q_1, q_2(m_2))$ for maintenance. Equilibrium choices are derived by combining both firms` reaction functions and solving equations for choice variables[10].

[10] The choices of refinery firms in equilibrium are expected to be symmetrical because they are assumed to be identical, i.e. have equal probability of failure functions, costs etc.

With assumed functional forms of probability and cost functions, the problem of firm 1 becomes:

$$\max_{q_1, m_1} \pi_1 = [(a + \frac{b}{q_1} + \frac{b}{q_2})q_1 - cq_1] \cdot (1 - \frac{q_1}{K_1} + \frac{m_1}{M_1})) - F \cdot (\frac{q_1}{K_1} - \frac{m_1}{M_1}) - gm_1^2 \quad (4)$$

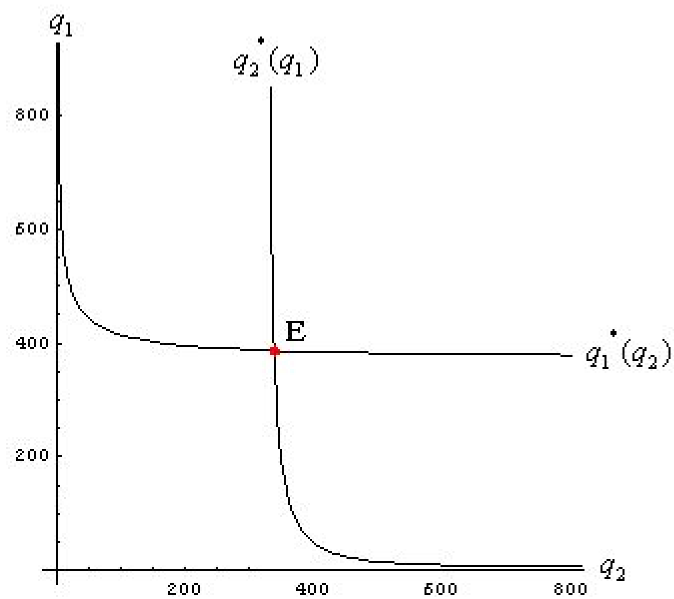
subject to: $q_1 \le K_1$ and $m_1 \le M_1$

After solving an equivalent problem for reaction functions of firm 2 analysis of simultaneous choice game comes down to four equations with four unknowns (see A2 in Appendix). The output and maintenance reaction functions of firm 1 are respectively:

$$q_1^*(q_2, m_1) = \frac{b(K_1(M_1 + m_1) - M_1 q_2) - q_2(FM_1 - K_1(a-c)(M_1 + m_1))}{2M_1(b + q_2(a-c))} \quad (5)$$

$$m_1^*(q_1, q_2) = \frac{q_2(F + q_1(a-c) + b) + bq_1}{2gM_1 q_2} \quad (6)$$

Simultaneous solution of (5) and (6) leads to a reaction function $q_1^*(q_2)$ (see A2 in Appendix). Using assumed values of parameters from Table 1 the products of refinery firms are seen to be strategic substitutes from the following graph:



Graph 2. Reaction curves determining equilibrium.

Reaction curves of refinery firms intersect at equilibrium point E (at this point $q_1^* = q_2^* \approx 386$ and $m_1^* = m_2^* \approx 6.37\ (\%)$). Maintenance reaction curve of firm 1 is depicted in Graph G29 in Appendix.

The resulting choice of production and maintenance levels states that firms in equilibrium will prefer to produce only at 40% of their capacity possibilities. This choice has a logical explanation: by doing so firms both decrease their need for more intensive maintenance (only about 7% of the total maintenance possible) and increase the price. These results are supported by additional simulations performed in Appendix, except for simulation of Table 6 in which refineries choose production level at more than 50% of their capacity limit and maintain 15% of the whole system[11].

Graphs G4-G9 in Appendix depict the variation of equilibrium output choice with all parameters of the model: for this purpose explanatory parameter varies in some range around its base value, while all other parameters are held constant at their base values. The resulting curves are pretty intuitive: it turns out that equilibrium choice of output increases only with an increase in capacity and parameters of price function[12].

Variation of equilibrium profits with output and maintenance choices are shown respectively in Graphs G30 and G31 in Appendix. These curves show the variation with one strategic variable while holding another constant (hence the maximum points in these graphs are not equilibrium values).

1.4.2 Sequential Move Competition

It is possible that in a regional market firms are not equal in timing of taking actions and hence make their choices one after another. With sequential move setting, the competition among firms

[11] Comparing to Table 1, Table 6 in Appendix assumes higher capacity and price, lower failure and production costs. This explains the choice of higher production level by refineries.
[12] It can be seen from Graph G9 in Appendix that the effect of an increase in *a* is much larger in magnitude than that of *b*. This is explained by specificity of price function and its elasticity coefficient.

consists of 2 stages: in the first stage first-moving refinery firm chooses its optimal maintenance and production levels; after observing the choice of its rival, second-moving firm decides on its production and maintenance levels in the second stage. Equilibrium outcome is found as usually by backward induction.

Assuming that refinery firm 1 moves first, derivation of optimal solutions for both firms starts with analysis of second stage (firm 2 actions). As before, maximizing profits with respect to q_2 and m_2 and solving first order conditions reaction function $q_2(q_1)$ is obtained for firm 2 (same as 11A in Appendix, but for q_2). Going back to first stage refinery firm 1 maximizes:

$$\max_{q_1, m_1} \pi_1 = [(a + \frac{b}{q_1} + \frac{b}{q_2(q_1)}) \cdot q_1 - cq_1] \cdot (1 - \frac{q_1}{K_1} + \frac{m_1}{M_1})) - F \cdot (\frac{q_1}{K_1} - \frac{m_1}{M_1}) - gm_1^2$$

subject to: $q_1 \leq K_1$ and $m_1 \leq M_1$

At this stage derivations are becoming laborious and there is a need for simulation. Using base values of parameters equilibrium output and maintenance are obtained for each firm: $q_1^* \approx 387.32$, $q_2^* \approx 385.54$, $m_1^* \approx 6.39$ (%), $m_2^* \approx 6.37$ (%). As was expected, first-moving refinery firm produces more output, but has a need for more maintenance. Graphs G10-G15 in Appendix show the dependence of optimal output choices on parameter values. The difference between two firms' choices decreases as capacity and maintenance cost increase.

Profits of competing firms are depicted in Graphs G16-G21 in Appendix. As maintenance cost increases, profit of first-moving refinery firm is approaching that of second moving firm. This is explained by the choice of first-moving firm – greater level of production demands greater level of maintenance and incurs higher maintenance costs.

Additional simulations in Appendix (p. 37-38) show the same results for different parameter values.

1.5 Concluding Remarks

In this Chapter interaction of refinery firms has been modeled as a static game, in which maintenance works and production have been assumed to be taken up simultaneously. This assumption can be considered as an unrealistic one for majority of maintenance works which demand stoppage of production. Nevertheless, the model has allowed identifying unique strategies of refinery firms that define Nash equilibrium.

Both simultaneous and sequential move competitions have showed similar results in simulations that used base values of parameters given in Table 1: it has turned out that it is better for each refinery firm to produce at about 40% of its capacity limit. This strategy has two benefits – low production (supply) implies higher price and decreases the need for more intensive maintenance. Sequential move game has showed that firm 1 chooses to produce slightly more than in simultaneous case, but still sticks to the optimal strategy of producing less than half of capacity possibilities.

Next Chapter develops this model further to a dynamic case which allows making more strict assumptions about maintenance and constructing interaction of refinery firms in a more realistic model.

Chapter 2: Dynamic Model and Repeated Games

In this chapter I develop previous model by introducing time periods. Such an extension has its benefits as well as drawbacks. The main advantage of this addition lies in the possibility of making more realistic assumptions. The refinery model of previous chapter has assumed that maintenance and production can be taken up at the same time. With the introduction of dynamic model it is feasible to make a reasonable assumption about maintenance being a time consuming event, i.e. refinery firms need to put the production off line in order to take up maintenance. In this context it will be possible to answer the question of whether it is optimal to take up the maintenance and stop the production or to continue producing no matter what.

Dynamic model has its disadvantage in comparison with static model in finding equilibrium. According to folk theorem, repeated games have a problem of multiple equilibria[13]. However the model will be extended to two periods only and equilibrium will still be defined as a unique one in this context. Hence the model allows analyzing the interaction of competing refineries in a more realistic construction.

2.1 Dynamic Programming

Dynamic programming is a method that allows solving a problem with overlapping sub-problems, i.e. a problem that can be divided into separate parts and optimized step-by-step. Here this problem is defined as maximizing total profits of refinery firms. A simple finite horizon model

[13] Folk Theorem states that sufficient condition for an outcome to be equilibrium in a repeated game is to satisfy minimax conditions, i.e. if the choice minimizes maximum possible loss for a player.

with 2 periods without discounting is analyzed. Profits of refinery firms are assumed to be time separable. Since one of the choice variables is assumed to be binary, it is impossible to use usual dynamic programming methods for solution and a simpler approach of comparison of the outcomes will be used.

For simulation purposes the following numbers will be used as base values:

K	F	c	G	a	b	φ	μ
1500	2000	3	500	10	200	0.25	0.1

Table 2. Base values of parameters used for simulations in dynamic model.

Other assumed values of parameters are used for additional simulations in Appendix (p. 39).

2.2 Single Firm Analysis

In this part of the Chapter demand is treated as fixed in order to determine the actions of refinery firm as a price taker. Dynamic programming allows putting restrictions on maintenance and production choices: maintenance decision will be treated as a binary choice variable with a fixed cost; if maintenance in some period is taken up, the production stops; if refinery decides to produce in some period it cannot take up maintenance. Basically the firm faces the problem of choosing between two options – either to produce or take up maintenance.

The cost of maintenance is fixed and equal to *G*. Firm starts making its choice in first period with initial $P(\text{failure}) = \varphi$. If in some period maintenance has been taken up, profits from production in that period are equal to zero and probability of failure decreases by some constant amount μ. Assuming that firm takes actions for two periods without discounting, it has two choices: either to take up maintenance in the first period and produce in the second or to produce in both periods[14]. Since maintenance decision is treated as a binary variable and puts restrictions on production, it would be impossible to analyze the choice of refinery firm by constructing a single profit function for both periods. Thus, 2 separate profit functions will be analyzed and compared:

$$\max\{\pi^1 + \pi^2\} = \begin{Bmatrix} \pi_m{}^1 + \pi_p{}^2 \\ \pi_p{}^1 + \pi_p{}^2 \end{Bmatrix} \tag{7}$$

where superscripts denote periods; π_m denotes profit of refinery firm if it takes up maintenance: $\pi_m = -G$ and $P(\text{failure}) = \varphi - \mu$; π_p denotes profit of refinery firm under production: $\pi_p = q \cdot (p^f - c) \cdot (1 - P(\text{failure})) - F \cdot P(\text{failure})$ with $P(\text{failure}) = \varphi + \frac{q}{K}$.

If refinery firm decides to produce in both periods, then its problem is:

$$\max_{q^1,q^2}\{\pi_p{}^1 + \pi_p{}^2\} = q^1(p^f - c)(1 - \varphi - \frac{q^1}{K}) - F(\varphi + \frac{q^1}{K}) + \\ + q^2(p^f - c)(1 - \varphi - \frac{q^1}{K} - \frac{q^2}{K}) - F(\varphi + \frac{q^1}{K} + \frac{q^2}{K})$$

The solution is found by first maximizing profits with respect to q^2 and deriving optimal q^2 as a function of q^1. After plugging it back into the profit function and maximizing with respect to q^1 maximized profit and optimal choices of refinery firm are found (see A3 in Appendix):

$$q^1 = \frac{K(1-\varphi)}{3} - \frac{F}{(p^f - c)} \qquad q^2 = \frac{K(1-\varphi)}{3}$$

$$\pi = \pi_p{}^1 + \pi_p{}^2 = \frac{F^2}{K(p^f - c)} + \frac{K(1-\varphi)^2(p^f - c)}{3} - F(1+\varphi)$$

[14] Since T=2 it would be meaningless to analyze cases when refinery firm takes up maintenance in the second period, because maintenance in the last period affects nothing but stoppage of production in that period.

In case the refinery decides to take up maintenance in the first period the problem becomes:

$$\max_{q^2}\{\pi_m^{\ 1}+\pi_p^{\ 2}\}=-G+q^2(p^f-c)(1-\varphi+\mu-\frac{q^2}{K})-F(\varphi-\mu+\frac{q^2}{K})$$

Maximizing output and optimal profit are obtained by solving first order conditions for q^2 of this problem (see A4 in Appendix):

$$q^2=\frac{K(1-\varphi+\mu)}{2}-\frac{F}{2(p^f-c)}$$

$$\pi=\pi_m^{\ 1}+\pi_p^{\ 2}=\frac{K(1-\varphi+\mu)^2(p^f-c)}{4}+\frac{F^2}{4K(p^f-c)}-\frac{F(1+\varphi-\mu)}{2}-G$$

It would be logical to conclude that if the refinery firm took up maintenance in the first period, it would produce more in the second period than it would without maintenance. This leads to the following condition:

$$\frac{K(1-\varphi+3\mu)}{3}>\frac{F}{(p^f-c)}$$

It can be seen that this condition may fail if initial probability of success and μ are low enough[15].

The problem of refinery firm now comes to comparison of profits under two different schemes. It is obvious that the choice of refinery firm between two schemes of actions in (7) mainly depends on initial probability of failure, μ and cost of maintenance *G*.

After plugging base values from Table 2 in both cases, it turns out that refinery firm enjoys higher profits after undertaking maintenance and producing more in second period ($\pi_m^{\ 1}+\pi_p^{\ 2}=341.80$, $\pi_p^{\ 1}+\pi_p^{\ 2}<0$). Parameter values from Tables 7,8,9 support this result, but

[15] This condition is satisfied for base values of parameters from Table 2.

using values from Table 10 changes the outcome in favor of producing in both periods ($\pi_m^{\ 1} + \pi_p^{\ 2} = 3440.63$, $\pi_p^{\ 1} + \pi_p^{\ 2} = 3992.50$). Parameter values from Table 10 decrease the need for maintenance by assuming: lower failure cost and higher maintenance cost; lower cost of production and higher capacity and price.

Graphs G22-G27 in Appendix show how overall equilibrium profits under both schemes change with all parameters. It is important to notice in G25 that under these base values profit of refinery firm when producing in both periods decreases to zero as initial probability of failure approaches the threshold value of about 33%. Graph G23 shows that equilibrium profits are higher under production in both periods when failure costs are low enough and the benefits of taking up maintenance decrease.

2.3 Duopoly Analysis

Demand function is assumed to be given in the same form as in previous Chapter. In Duopoly case each firm has a choice in form of (7). If one firm decides to take up maintenance in the first period, the other firm gets all the residual demand. I will analyze interaction of refinery firms again under simultaneous and sequential move games, later introducing uncertainty.

2.3.1 Simultaneous Move Competition

Using assumed functional forms for costs and probability function, each firm maximizes sum of its time-separable profits. As before, each refinery decides whether to take up maintenance in the first period or to skip it. In case of duopoly this means that there are 4 possible states in which firms may end up: both firms always produce; both firms take up maintenance in the first period; one of the firms takes up maintenance in the first period, the other supplies the market. Optimal solutions are obtained by backward induction for each case.

Again, since maintenance is a binary choice variable it is impossible to analyze actions of refinery firms by constructing a single production function, and hence all cases should be compared separately using simulations. For the case when both refineries produce in both periods, the problem of firm i ($i = 1,2$) is:

$$\max_{q_i^1, q_i^2}\{\pi_i^1 + \pi_i^2\} = q_i^1(a + \frac{b}{q_i^1} + \frac{b}{q_{-i}^1} - c)(1 - \varphi - \frac{q_i^1}{K}) - F(\varphi + \frac{q_i^1}{K}) + \\ + q_i^2(a + \frac{b}{q_i^2} + \frac{b}{q_{-i}^2} - c)(1 - \varphi - \frac{q_i^1}{K} - \frac{q_i^2}{K}) - F(\varphi + \frac{q_i^1}{K} + \frac{q_i^2}{K}) \quad (8)$$

where subscript "$-i$" denotes rival firm and superscripts denote periods. The solution is symmetrical and is obtained using backward induction - starting from optimization of period 2 output choice and going back to period 1. When both firms decide to take up maintenance, they compete only in second period and the problem of firm i becomes:

$$\max_{q_i^2}\{\pi_i^1 + \pi_i^2\} = -G + q_i^2(a + \frac{b}{q_i^2} + \frac{b}{q_{-i}^2} - c)(1 - \varphi + \mu - \frac{q_i^2}{K_i}) - F(\varphi - \mu + \frac{q_i^2}{K_i}) \quad (9)$$

As for the separating case (one refinery produces in both periods, another takes up maintenance in the first period) the producing refinery i receives all the residual demand in the first period and the price is determined as: $p^1 = a + \frac{b}{q_i^1}$. The solution is again derived by backward induction. The resulting profits and output choices are presented in the following 2x2 payoff matrix:

Firm 1 \ Firm 2	Produce-Produce	Maintain-Produce
Produce-Produce	$\pi_1 = 317.321$ $q_1^1 = 32.15$ $q_1^2 = 375.21$ $\pi_2 = 317.321$ $q_2^1 = 32.15$ $q_2^2 = 375.21$	$\pi_1 = 137.984$ $q_1^1 = 39.49$ $q_1^2 = 394.30$ $\pi_2 = 513.056$ $q_2^1 = 0$ $q_2^2 = 603.51$
Maintain-Produce	$\pi_1 = 513.056$ $q_1^1 = 0$ $q_1^2 = 603.51$ $\pi_2 = 137.984$ $q_2^1 = 39.49$ $q_2^2 = 394.30$	$\pi_1 = 551.245$ $q_1^1 = 0$ $q_1^2 = 489.03$ $\pi_2 = 551.245$ $q_2^1 = 0$ $q_2^2 = 489.03$

where "Produce-Produce" stands for producing in both periods and "Maintain-Produce" - for taking up maintenance in the first period. This game may seem similar to Prisoner`s Dilemma, however it is obvious from this matrix that Maintain-Produce is a dominant strategy for both firms.

Optimal choices of output levels state the strategies of refinery firms. It turns out that if a firm decides to produce in both periods, it prefers to keep the production on low level in the first one to compete more harshly in the second. Much higher level of production in the second period is also explained by the limitation of model to two periods: the choice of output in the last period can increase the probability of failure in that period, but it does not affect the future probability of failure since the game is terminated at that point. However extension of the model to T periods is predicted to yield the same results: firms will produce on low levels until the last period and in period T the production level will greatly increase.

It is not surprising that refinery firms still prefer to follow the strategy of low production and zero maintenance if both firms choose to always produce. However in case when one of the refineries decides to take up maintenance, the other seems to be insensitive to the residual demand and still chooses to produce on low (but slightly higher) level because it anticipates that the rival firm will come back with much stronger possibilities to produce in the second period. Hence, refinery firm prefers to be prepared for the competition in the last period by producing less in the first. In current model refineries seem to be very sensitive to the probability of failure and give up additional profits from residual demand for safer future production.

It is also worth noticing that the consumers are always better off when refineries take up maintenance: the total supply is higher and the price is lower. Even though the best option for the consumers is when one of the refineries is always producing and another takes up maintenance[16], refinery firms arrive at an equilibrium when both take up maintenance and enjoy higher overall profits.

Additional simulations performed in Appendix (p.39) show that using parameter values from Tables 8 and 10 makes production in both periods a dominant strategy for both refineries. This change shows that in a dynamic model choices of refineries are sensitive to market price of product, cost of production and the amount of a decrease in probability of failure after maintenance works (μ).

2.3.2 Sequential Move Competition

Under sequential move construction equilibrium outcome does not change: first-moving refinery firm anticipates that it will not affect the choice of the second-moving firm, since taking up

[16] This case is better than the one when both firms choose to take up maintenance, because in the former the total supply is lower and the market is compelled to supply the consumers in the first period with expensive imported products.

maintenance is a dominant strategy for both firms. This conclusion is resulting from the choice of specific values of parameters from Table 2. It is possible to readjust them in such a way that taking up maintenance is no longer a dominant strategy. Additional simulations in Appendix (p. 39) have showed that the equilibrium outcome changes to production in both periods after assuming: higher price and maintenance cost; lower μ and production cost.

The next subsection introduces uncertainty and demonstrates how equilibrium depends on beliefs of refinery firms about demand and shows an example when producing in both periods becomes a dominant strategy.

2.4 Introducing Uncertainty

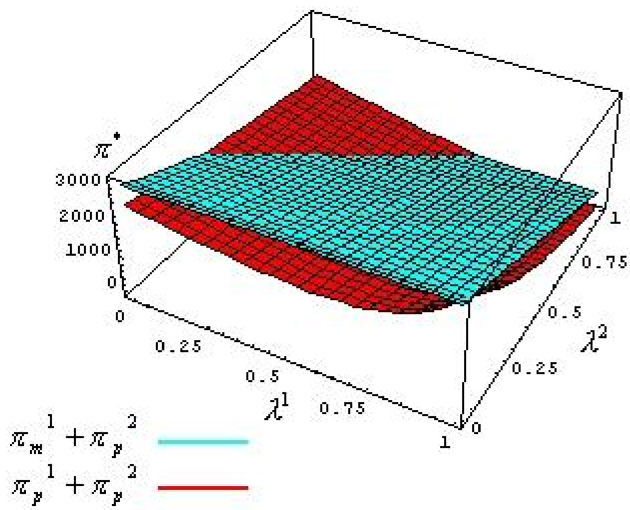
In a world of perfect information choices of refinery firms are based on their knowledge about costs and demand. This part of the Chapter extends the previous model by introducing uncertainty about demand and hence – price of the product. Firms are assumed to have identical beliefs about the future price: the demand will be low with probability λ^1 in the first period and with probability λ^2 in the second. Therefore the problems of refineries are unchanged with the only difference in their beliefs about price. Using probabilities that refinery firms put on low demand, prices in first and second period respectively are expected to be:

$$p^1 = \lambda^1 \cdot (a_L + \frac{b_L}{q_1^1} + \frac{b_L}{q_1^1}) + (1-\lambda^1) \cdot (a_H + \frac{b_H}{q_1^1} + \frac{b_H}{q_1^1})$$

$$p^2 = \lambda^2 \cdot (a_L + \frac{b_L}{q_1^2} + \frac{b_L}{q_1^2}) + (1-\lambda^2) \cdot (a_H + \frac{b_H}{q_1^2} + \frac{b_H}{q_1^2})$$

where subscripts L and H stand for Low and High respectively and: $a_L < a_H$, $b_L < b_H$. The results obtained without uncertainty do not change if $\lambda^1 = \lambda^2$: that is, if refinery firm believes that demand will be high or low with the same probability in each period, it will still stick to its previous strategy of taking up maintenance.

The following graph shows how maximized profits change with beliefs (probabilities) about demand for a single price-taking firm case when $p^1 = \lambda^1 p_L + (1-\lambda^1) p_H$ and $p^2 = \lambda^2 p_L + (1-\lambda^2) p_H$:



Graph 3. Optimal profits changing with beliefs (probabilities of low demand)

It can be seen from Graph 3 that producing in both periods can be optimal for a refinery firm if demand is likely to be high in the first period and low in the second ($\lambda^1 < 0.5$ and $\lambda^2 > 0.5$). It is also obvious that taking up maintenance is always better when $\lambda^1 \geq \lambda^2$, i.e. when demand in the first period is expected to be low with a probability at least as great as probability of low demand in the second period.

Application of uncertainty to interaction of refinery firms can show whether it will significantly affect their choices of output and profits. The general derivation of profits as functions of beliefs is laborious and unintuitive. I use simulation with base values from Table 2 and some additional values stated later to simplify the analysis.

The following example is an application of uncertainty to simultaneous move interaction of refinery firms with the following additional assumptions on the values of parameters: $a_H = 20$, $b_H = 500$, $a_L = 10$, $b_L = 200$, $\lambda^1 = 0$, $\lambda^2 = 0.5$. The assumptions on probabilities state that demand is definitely going to be high in the first period, but in the second period there is a 50% probability of a low demand. The resulting payoff matrix is:

Firm 2 / Firm 1	Produce-Produce	Maintain-Produce
Produce-Produce	$\pi_1 = 2661.670$ $q_1^1 = 286.61$ $q_1^2 = 306.70$ $\pi_2 = 2661.670$ $q_2^1 = 286.61$ $q_2^2 = 306.70$	$\pi_1 = 2332.820$ $q_1^1 = 301.20$ $q_1^2 = 318.93$ $\pi_2 = 2117.602$ $q_2^1 = 0$ $q_2^2 = 547.79$
Maintain-Produce	$\pi_1 = 2117.602$ $q_1^1 = 0$ $q_1^2 = 547.79$ $\pi_2 = 2332.820$ $q_2^1 = 301.20$ $q_2^2 = 318.93$	$\pi_1 = 1996.94$ $q_1^1 = 0$ $q_1^2 = 544.56$ $\pi_2 = 1996.94$ $q_2^1 = 0$ $q_2^2 = 544.56$

It is obvious that in this case producing in both periods is a dominant strategy for each firm. Under this construction equilibrium outcome is also the best for consumers which enjoy highest total supply and lowest average price.

2.5 Concluding Remarks

This Chapter has developed a simple but useful dynamic model of interaction of refinery firms. Dynamic approach has allowed me to analyze the actions of refinery firms under more realistic assumptions about maintenance decisions: production and maintenance works have been assumed to be exclusive decision variables.

The results derived with the help of simulations have identified the optimal strategies of refinery firms. Construction of 2x2 payoff matrix has showed that taking up maintenance is a dominant strategy under assumed values of parameters. In cases when refinery decides to produce in both periods, it should stick to the strategy derived in the previous Chapter: produce less and enjoy higher prices with lower probability of failure. The model has also demonstrated insensitivity of refinery firms to residual demand and high sensitivity to probability of failure.

The surprising difference between optimal production choice in the first and second periods comes from the fact that the choice game is terminated after production in the second period. Extension of the model to more periods should not change the general result of this outcome, because the firms will still produce at an optimal low level until the last period and boost the production right when the game terminates and probability of failure is no longer affected.

Later introducing uncertainty the model has showed that the equilibrium outcome depends on beliefs of refinery firms about demand. The result has turned out pretty intuitive: when demand is expected to be high in the first period and low in the second, it is optimal to produce in both periods. This change of equilibrium from maintenance to production is also welcomed by customers because they enjoy higher total supply and lower average price.

Conclusions

Harsh competition and a constantly changing demand for oil require refineries to produce at full capacity possibilities and create bad incentives to skip production stoppage for maintenance works. This, in turn, can lead to a system failure, which creates a danger of explosions (fires) and incurs huge costs of restoration of refining machinery. In my thesis, I have analyzed the choices of production and maintenance levels chosen by refinery firms. Besides defining profits, production and maintenance were assumed to be strategic decisions which affect the probability of failure.

Using classical game theory tools and simulations of parameters, I have identified the optimal strategies for refinery firms in static and dynamic models under different situations: single refinery firm being a price taker; competition of two refinery firms that make their choices of strategic variables` levels simultaneously; competition of two refinery firms deciding on strategic variables sequentially, one after another.

Static model in Chapter 1 has allowed optimizing refineries` optimal (simultaneous) choices of output and maintenance levels in a convenient and simple way. In this model, the result of interaction of the refinery firms under different constructions (simultaneous and sequential moves) has turned out to be surprising but intuitive: in all cases, refineries have been involved in a kind of tacit collusion by supplying the regional market with considerably less quantity of refined products than possible. By doing so, refinery firms have been enjoying higher prices and have decreased their need for maintenance (less than 10% of total refinery system demands maintenance in this equilibrium). The advantage of the static model lies in the simplicity and the possibility of defining a unique equilibrium. However, this model makes it impossible to put realistic restrictions on the choices of maintenance and production. The majority of maintenance works in oil refinery sector demands production stoppage for the period of maintenance, mainly because of complex interconnection of all parts of the refinery system.

Introduction of dynamic model in Chapter 2 has allowed for necessary restrictions: maintenance has been assumed to be a binary variable with fixed costs of implementation and fixed fraction of a decrease in probability of failure; production and maintenance have been exclusive events. Despite this advantage, dynamic models and repeated games come with their problem of multiple equilibria. Intuitively, this means that in an infinite horizon model, both firms can end up in any of the situations when they are both better off. This disadvantage, however, has been overcome by the introduction of finite horizon (2 periods) model. The results of the dynamic model have showed that refineries are sensitive to any possibilities of failure and both of the competing refineries end up taking up maintenance in the first period. In choosing production levels, firms still follow the optimal strategy of static model: producing less and enjoying safety with higher prices. Later, extending the model with uncertainty, I have showed that decisions of refineries can switch to production in both periods depending on their beliefs about demand.

Both models presented in my thesis are simple but informative. Further extensions can help to understand the interaction and choices of oil refineries. Among possible ones, I would suggest: analyzing in static model the interaction of non-identical refineries that differ in their capacities, market power etc.; extending dynamic model to more periods (finite) or infinite horizon and studying more in-depth decisions of refineries on optimal frequency of maintenance; testing both dynamic and static models empirically on whether the equilibrium outcome is supported by data.

Appendix

Static Model

A1. Single Firm Choice with Fixed Demand

Using assumed functional forms of probability and cost functions, firm maximizes its profit with respect to output and maintenance:

$$\frac{\partial \pi}{\partial q_i} = (1 - \frac{q_i}{K_i} + \frac{m_i}{M_i}) \cdot [p^f - c] - \frac{q_i \cdot (p^f - c)}{K_i} - \frac{F}{K_i} = 0 \qquad (1A)$$

$$\frac{\partial \pi}{\partial m_i} = \frac{(p^f - c) \cdot q_i + F}{M_i} - 2gm_i = 0 \qquad (2A)$$

Solving (1A) and (2A) for *q* and *m* respectively brings the problem to:

$$q_i^* = \frac{K_i(m_i + M_i)}{2M_i} - \frac{F}{2(p^f - c)} \qquad (3A)$$

$$m_i^* = \frac{(p^f - c) \cdot q_i + F}{2gM_i} \qquad (4A)$$

Profit maximizing output and maintenance choices are found in terms of parameters by solving simultaneously equations (3A) and (4A).

Dependence of optimal maintenance and output choices on different parameters is found by differentiation:

$$\frac{\partial m_i^*}{\partial K_i} = \frac{M_i(4gM_i^2 + F)(p^f - c)}{(4gM_i^2 - K_i(p^f - c))^2} \Rightarrow \frac{\partial m_i^*}{\partial K_i} > 0$$

$$\frac{\partial q_i^*}{\partial K_i} = \frac{2gM_i^2(4gM_i^2 + F)}{(4gM_i^2 - K_i(p^f - c))^2} \Rightarrow \frac{\partial q_i^*}{\partial K_i} > 0$$

$$\frac{\partial m_i^*}{\partial M_i} = -\frac{(4gM_i^2 + K_i(p^f - c))(F + K_i(p^f - c))}{(4gM_i^2 - K_i(p^f - c))^2} \Rightarrow \frac{\partial m_i^*}{\partial M_i} < 0$$

$$\frac{\partial q_i^*}{\partial M_i} = -\frac{4gK_iM_i(F+K_i(p^f-c))}{(4gM_i^2-K_i(p^f-c))^2} \quad \Rightarrow \quad \frac{\partial q_i^*}{\partial M_i} < 0$$

$$\frac{\partial m_i^*}{\partial F} = \frac{M_i}{4gM_i^2-K_i(p^f-c)} \quad \Rightarrow \quad \frac{\partial m_i^*}{\partial F} > 0 \quad \Leftrightarrow \quad 4gM_i^2 > K_i(p^f-c)$$

(5A)

$$\frac{\partial q_i^*}{\partial F} = -\frac{2gM_i^2-K_i(p^f-c))}{(p^f-c)(4gM_i^2-K_i(p^f-c))} \quad \Rightarrow \quad \frac{\partial q_i^*}{\partial M_i} < 0 \quad \Leftrightarrow \quad 2gM_i^2 > K_i(p^f-c))$$

Last condition is necessary for the optimal output to decrease with F and is sufficient for optimal maintenance choice to increase with F.

A2. Simultaneous Choice Game in Duopoly

First order conditions for the problem are:

$$\frac{\partial \pi}{\partial q_1} = (1-\frac{q_1}{K_1}+\frac{m_1}{M_1})\cdot[a-c+\frac{b}{q_2}] - \frac{q_1[a-c+\frac{b}{q_1}+\frac{b}{q_2}]}{K_1} - \frac{F}{K_1} = 0 \qquad (7A)$$

$$\frac{\partial \pi}{\partial m_1} = \frac{q_1[a-c+\frac{b}{q_1}+\frac{b}{q_2}+F]}{M_1} - 2gm_1 = 0 \qquad (8A)$$

The first one leads to the output reaction function of firm 1:

$$q_1^*(q_2, m_1) = \frac{b(K_1(M_1+m_1)-M_1q_2)-q_2(FM_1-K_1(a-c)(M_1+m_1))}{2M_1(b+q_2(a-c))} \qquad (9A)$$

FOC for maintenance level defines optimal choice of maintenance as:

$$m_1^*(q_1, q_2) = \frac{q_2(F+q_1(a-c)+b)+bq_1}{2gM_1q_2} \qquad (10A)$$

Reaction function $q_1^*(q_2)$ is obtained by solving (9A) and (10A) simultaneously for optimal choice of output:

$$q_1^*(q_2)=\frac{q_2(b^2K_1+q_2(K_1(a-c)(F+2gM_1^{\ 2})-2FgM_1^{\ 2})+b(FK_1+2gM_1^{\ 2}(K_1-q_2)+K_1q_2(a-c)))}{(b+q_2(a-c))(q_2(4gM_1^{\ 2}-K_1(a-c))-bK_1)} \quad (11A)$$

Equilibrium outcome is found by solving the same problem for firm 2 in a similar way and combining the 2 reaction functions for output of both firms. The solution is burdensome and has no intuitive value without using simulations.

Dynamic Model

A3. Single Firm Choice with Fixed Demand

When the firm decides to produce in both periods, the solution is found by backward induction. Maximizing profit function with respect to q^2, the firm obtains:

$$\frac{\partial(\pi_p^{\ 1}+\pi_p^{\ 2})}{\partial q^2}=(p-c)(1-\varphi-\frac{q^1+2q^2}{K})-\frac{F}{K}=0 \quad \Rightarrow \quad q^2=\frac{K(1-\varphi)-q^1}{2}-\frac{F}{2(p-c)}$$

Plugging it back to profit function and maximizing with respect to q^1:

$$\frac{\partial(\pi_p^{\ 1}+\pi_p^{\ 2})}{\partial q^1}=\frac{1}{2K}[(p-c)(K(1-\varphi)-3q^1)-3F]=0 \quad \Rightarrow \quad q^1=\frac{K(1-\varphi)}{3}-\frac{F}{(p-c)}$$

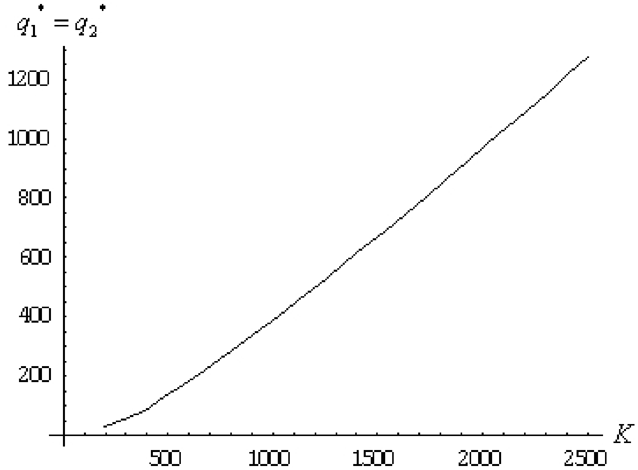
Maximized profit is obtained by solving for optimal q^2 using q^1 and finally - using both in profit function.

For the case when a firm decides to take up maintenance first, optimal output choice in the second period is found from FOC:

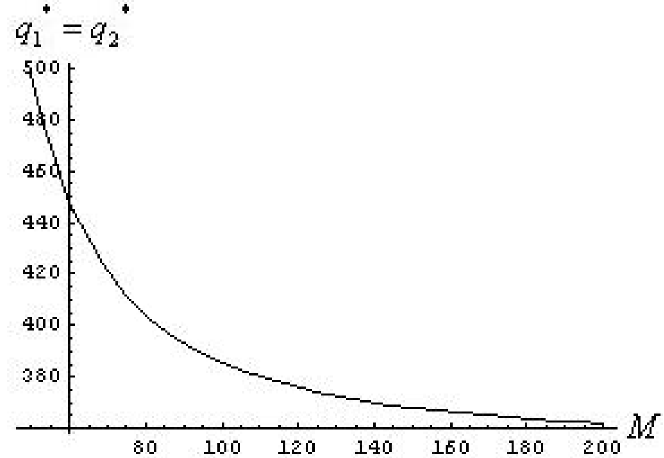
$$\frac{\partial(\pi_m^{\ 1}+\pi_p^{\ 2})}{\partial q^2}=(p-c)(1-\varphi+\mu-\frac{2q^2}{K})-\frac{F}{K}=0 \quad \Rightarrow \quad q^2=\frac{K(1-\varphi+\mu)}{2}-\frac{F}{2(p-c)}$$

The final result is obtained by plugging optimal q^2 back into profit function.

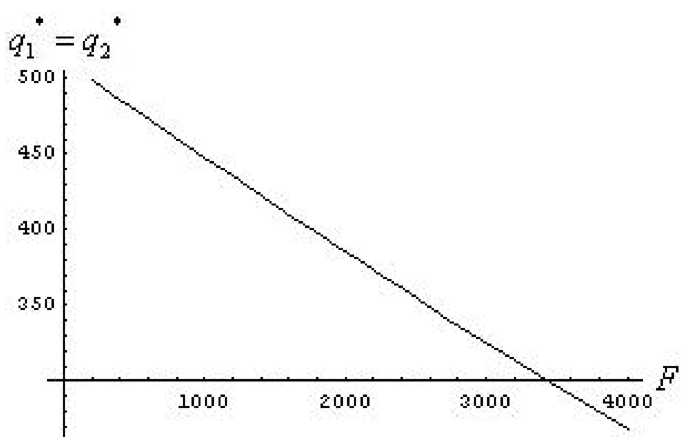
Static Model – Simultaneous Moves



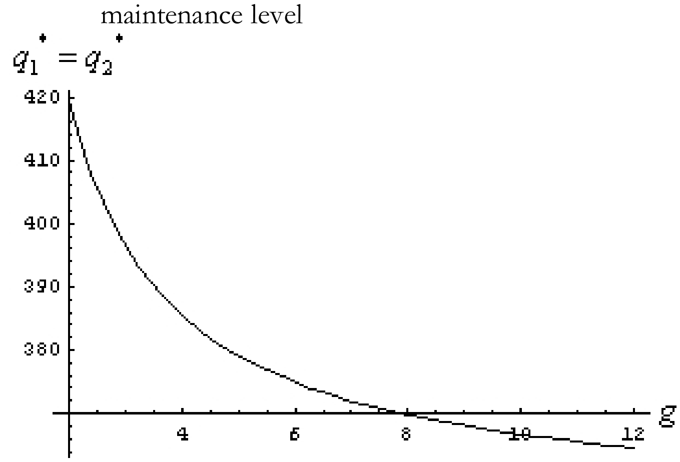
G4. Variation of equilibrium output with capacity



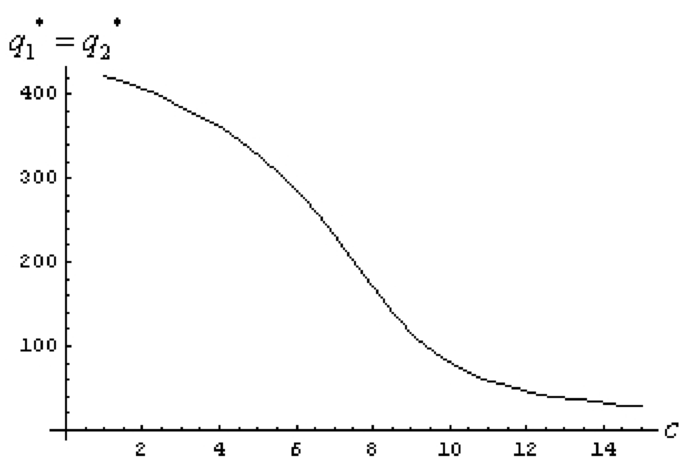
G5. Variation of equilibrium output with maximum maintenance level



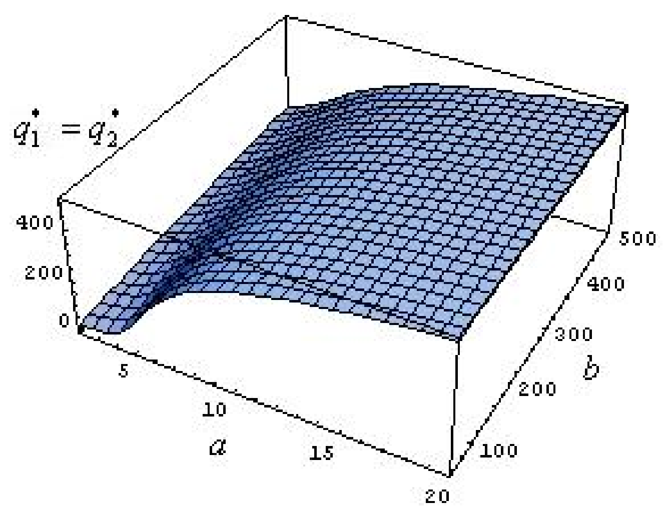
G6. Variation of equilibrium output with failure cost



G7. Variation of equilibrium output with maintenance Cost

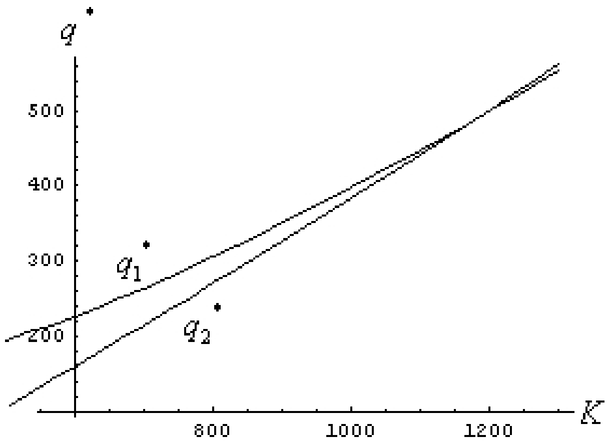


G8. Variation of equilibrium output with production cost

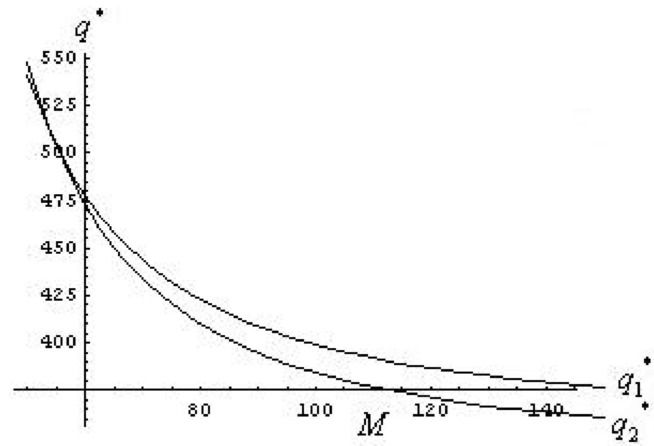


G9. Variation of equilibrium output with price function

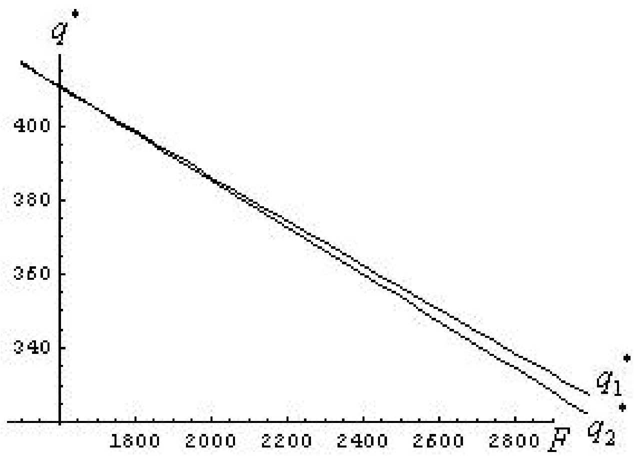
Static Model - Sequential Moves



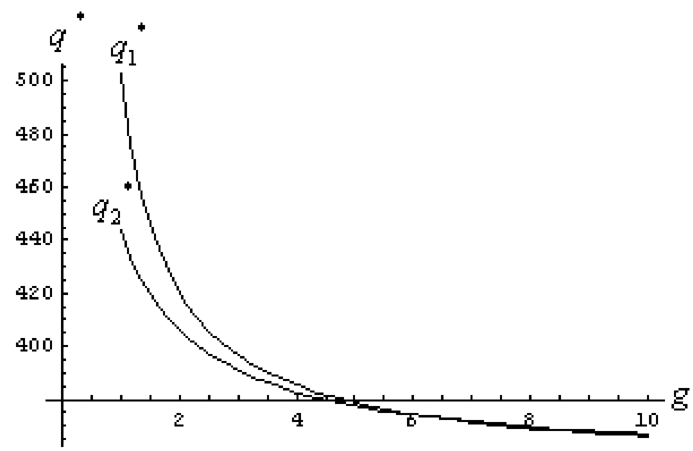
G10. Variation of equilibrium outputs with capacity



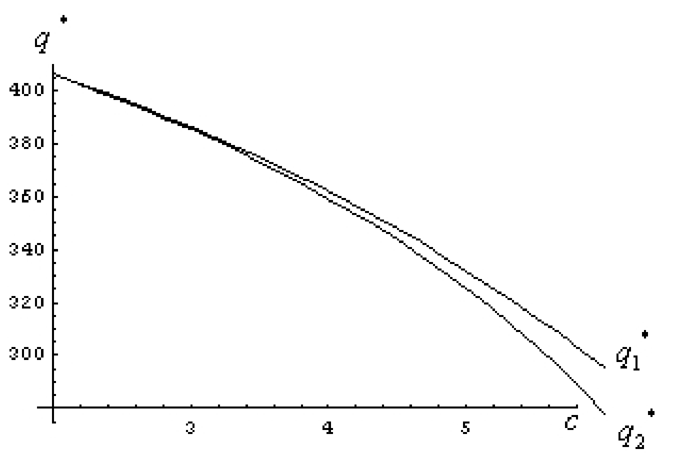
G11. Variation of equilibrium outputs with maximum maintenance level



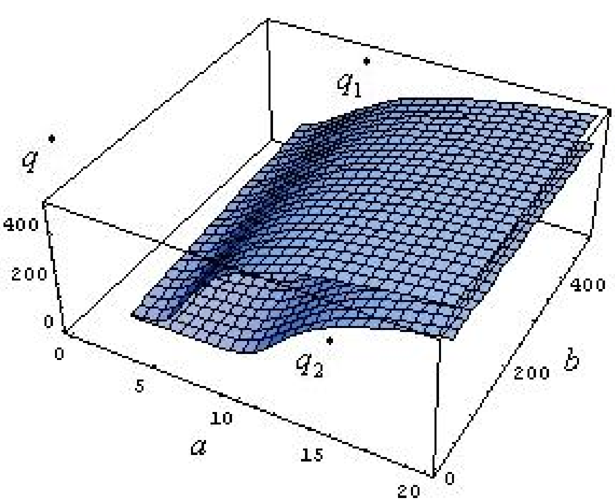
G12. Variation of equilibrium outputs with failure cost



G13. Variation of equilibrium outputs with maintenance cost

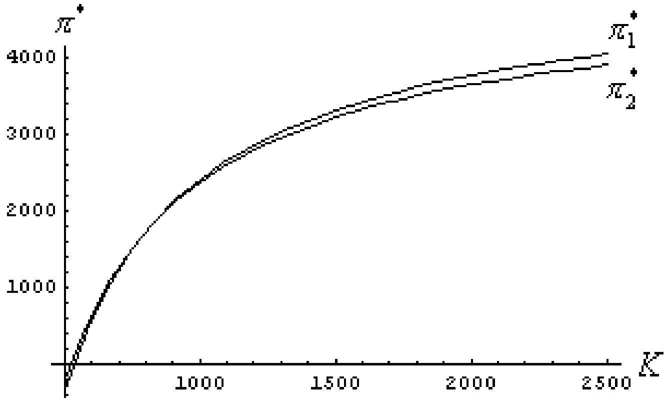


G14. Variation of equilibrium outputs with production cost

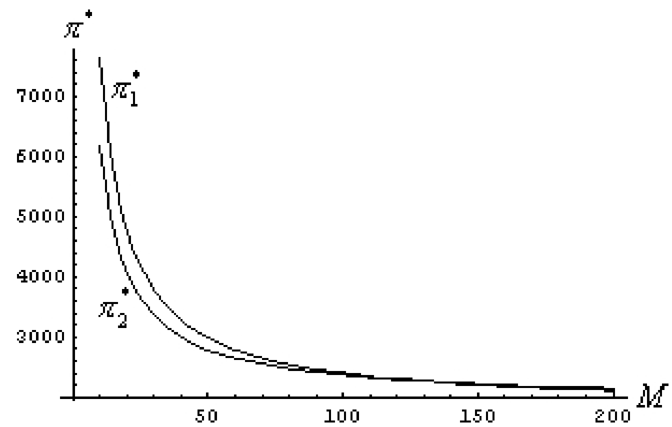


G15. Variation of equilibrium outputs with price function

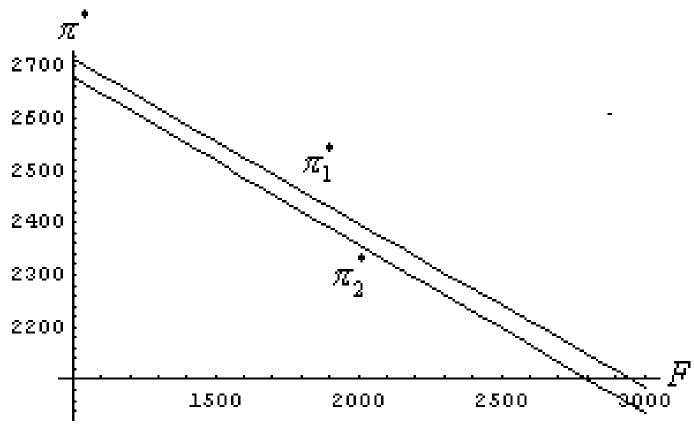
Static Model - Sequential Moves



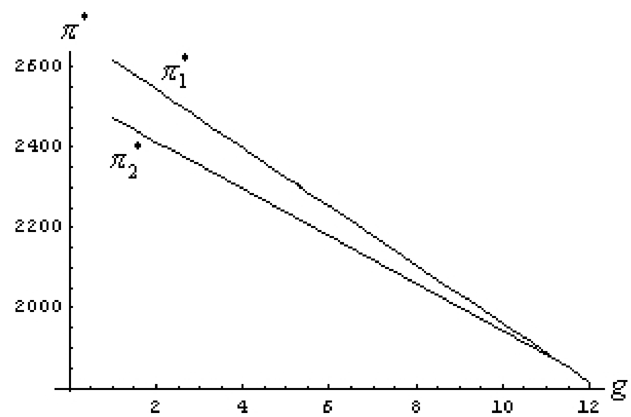
G16. Variation of equilibrium profits with capacity



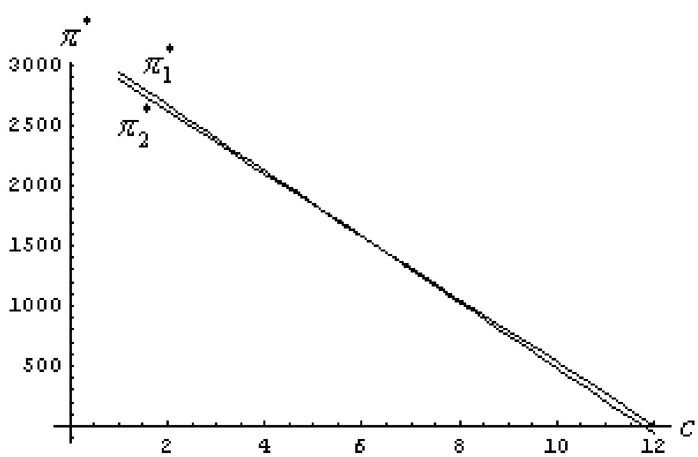
G17. Variation of equilibrium profits with maximum maintenance level



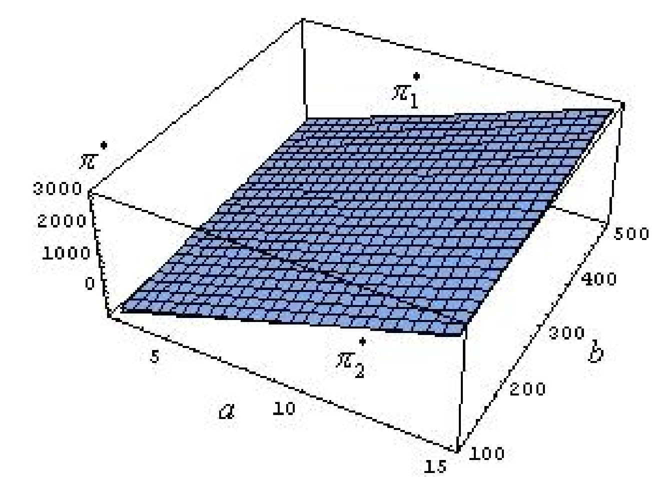
G18. Variation of equilibrium profits with failure cost



G19. Variation of equilibrium profits with maintenance cost

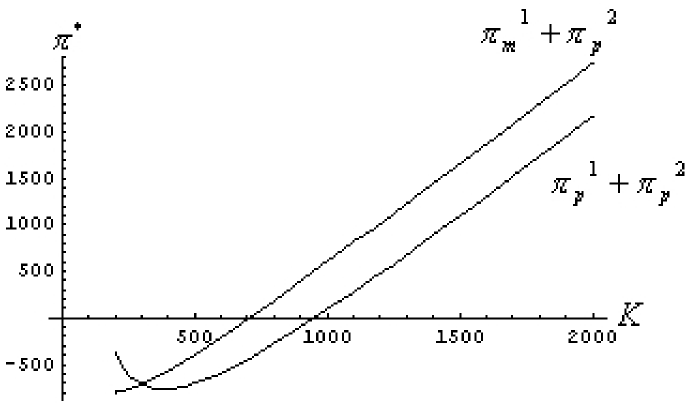


G20. Variation of equilibrium profits with production cost

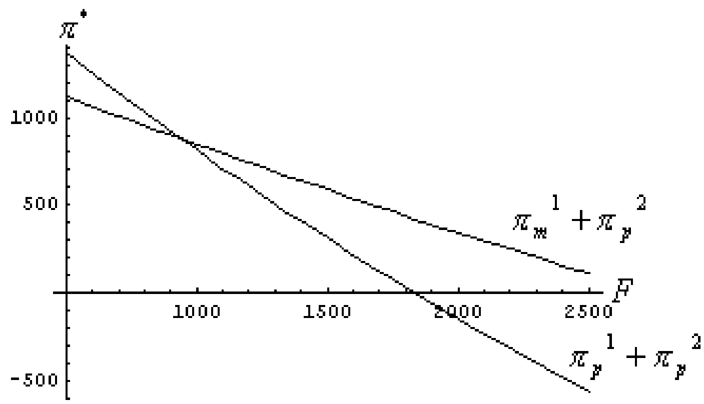


G21. Variation of equilibrium profits with price function

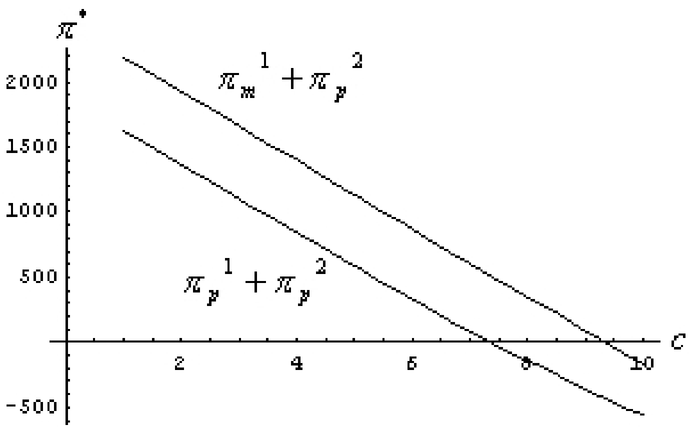
Dynamic Model – Single Firm



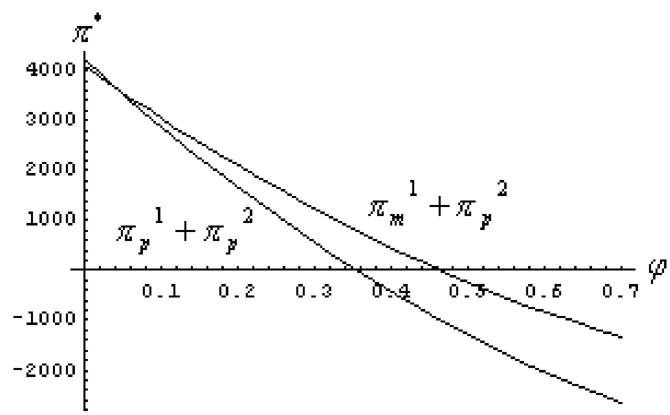
G22. Variation of equilibrium profits with capacity



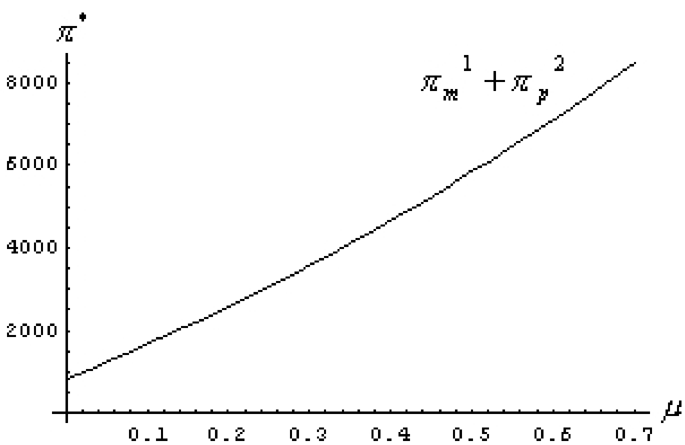
G23. Variation of equilibrium profits with failure cost



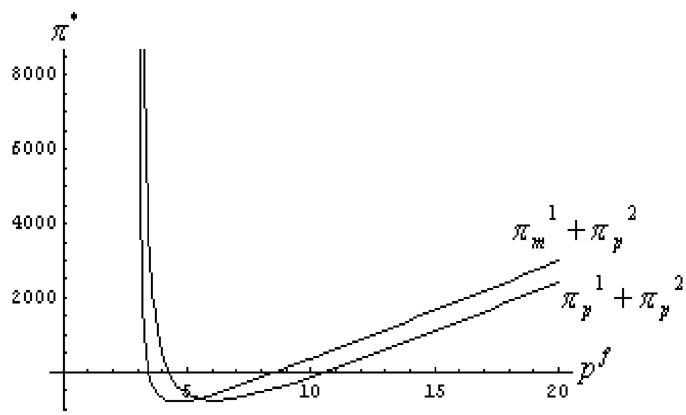
G24. Variation of equilibrium profits with production costs



G25. Variation of equilibrium profits with initial probability of failure

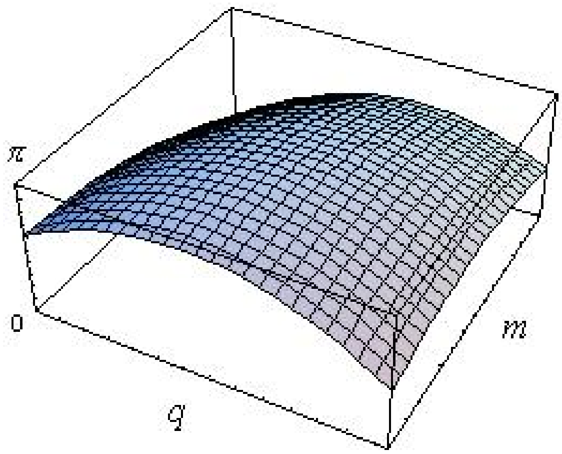


G26. Variation of equilibrium profits with a decrease in probability of failure due to maintenance

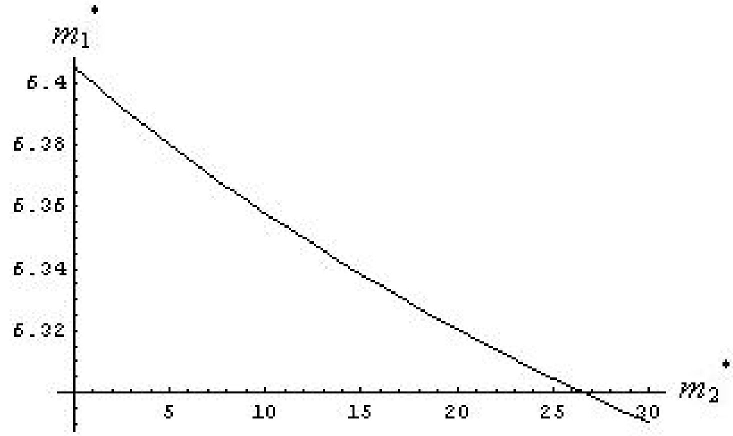


G27. Variation of equilibrium profits with fixed price

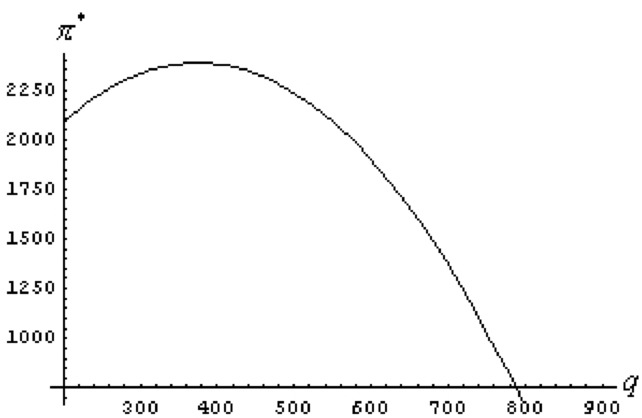
Miscellaneous



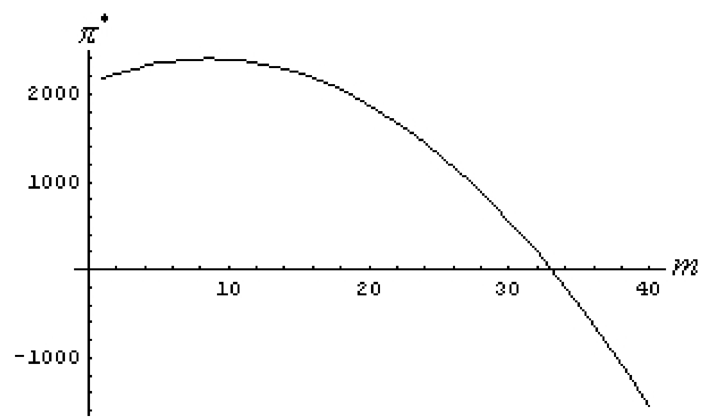
G28. Variation of profit with output and maintenance in single firm case



G29. Maintenance reaction curve in simultaneous move game



G30. Variation of equilibrium profit with output choice in simultaneous move game



G31. Variation of equilibrium profit with maintenance choice in simultaneous move game

Additional Simulations – Static Model

K	*M*	*F*	*c*	*g*	*a*	*b*
500	100	2500	3	4	10	200

Table 3

K	M	F	c	g	a	b
500	100	2500	2	6	20	500

Table 4

K	M	F	c	g	a	b
1500	100	1500	3	4	10	200

Table 5

K	M	F	c	g	a	b
1500	100	1500	2	6	20	500

Table 6

Type of Game	Simultaneous Moves	Sequential Moves	
Firm / Values	Firms 1 & 2	First-mover (Firm 1)	Second-mover (Firm 2)
Table 3	$q_1^* = q_2^* = 108.72$ $m_1^* = m_2^* = 4.58$ (%) $\pi_1^* = \pi_2^* = 448.76$	$q_1^* = 122.98$ $m_1^* = 4.74$ (%) $\pi_1^* = 452.11$	$q_2^* = 104.76$ $m_2^* = 4.50$ (%) $\pi_2^* = 429.81$
Table 4	$q_1^* = q_2^* = 191.69$ $m_1^* = m_2^* = 5.79$ (%) $\pi_1^* = \pi_2^* = 1987.05$	$q_1^* = 192.94$ $m_1^* = 5.81$ (%) $\pi_1^* = 1987.12$	$q_1^* = 191.62$ $m_1^* = 5.79$ (%) $\pi_1^* = 1984.87$
Table 5	$q_1^* = q_2^* = 696.9$ $m_1^* = m_2^* = 8.47$ (%) $\pi_1^* = \pi_2^* = 2416.26$	$q_1^* = 697.10$ $m_1^* = 8.47$ (%) $\pi_1^* = 2416.26$	$q_1^* = 696.90$ $m_1^* = 8.46$ (%) $\pi_1^* = 2416.23$
Table 6	$q_1^* = q_2^* = 802.17$ $m_1^* = m_2^* = 14.12$ (%) $\pi_1^* = \pi_2^* = 7575.94$	$q_1^* = 802.30$ $m_1^* = 14.12$ (%) $\pi_1^* = 7575.94$	$q_1^* = 802.17$ $m_1^* = 14.11$ (%) $\pi_1^* = 7575.89$

Additional Simulations – Dynamic Model

K	F	c	G	a	b	φ	μ
1000	2500	3	300	10	200	0.2	0.15

Table 7

K	F	c	G	a	b	φ	μ
1000	2500	2	700	20	500	0.3	0.05

Table 8

K	F	c	G	a	b	φ	μ
2000	1500	3	300	10	200	0.2	0.15

Table 9

K	F	c	G	a	b	φ	μ
2000	1500	2	700	20	500	0.3	0.05

Table 10

Choice / Values	F1 Produce-Produce F2 Produce-Produce	F1 Maintain-Produce F2 Produce-Produce	F1 Maintain-Produce F2 Maintain-Produce
Table 7	$\pi_1^* = \pi_2^* = 14.76$	$\pi_1^* = 450.46$; $\pi_1^* < 0$	$\pi_1^* = \pi_2^* = 457.23$
Table 8	$\pi_1^* = \pi_2^* = 1070.56$	$\pi_1^* = 800.72$; $\pi_1^* = 701.87$	$\pi_1^* = \pi_2^* = 813.89$
Table 9	$\pi_1^* = \pi_2^* = 1764.52$	$\pi_1^* = 2072.46$; $\pi_1^* = 1603.34$	$\pi_1^* = \pi_2^* = 2325.71$
Table 10	$\pi_1^* = \pi_2^* = 4803.61$	$\pi_1^* = 3466.16$; $\pi_1^* = 4489.11$	$\pi_1^* = \pi_2^* = 3843.40$

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